Modelling “pantographic beams” by means of a nonlinear 1D second gradient continuum model

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In the presentation we consider a so-called “pantographic beam”. We introduce a mechanical discrete spring Hencky-type micro-model which resembles the micro-structure of the beam. Extensional and rotational spring with their associated potentials, aimed at taking into account extension and bending of each fiber, are introduced. Then, in order to formulate a continuum fully nonlinear beam model, we propose a heuristic homogenization technique with proper rescaling laws for the rigidities of all the springs of the micro-model and for the length of each fiber. We show that the problem of minimizing the homogenized internal strain energy can be formulated relying only upon the placement field of the 1D continuum i.e. a “reduced” internal strain energy depending on the placement field while not on the length of each fiber (which can be explicitly expressed at equilibrium as a function of placement) is derived. Finally, interesting features of the 1D continuum model are discussed along with its consistency with the micro-model from which it is derived. The internal strain energy of the homogenized 1D continuum model will be then discussed for different spring configurations not allowing for some notable floppy modes. Particular emphasis will be given to the case in which the fibers constituting the micro-structure are inextensible and, in this case, connections with previous literature will be traced.