A theory of remodelling in fibre-reinforced anisotropic media based on the Ginzburg-Landau free energy density

Alfio Grillo[1], **Melania Carfagna**[1], **Salvatore Federico**[2]

[1]Department of Mathematical Sciences (DISMA) "G.L. Lagrange", Politecnico di Torino

Corso Duca degli Abruzzi 24, 10129, Torino, Italy. E-mail: {alfio.grillo melania.carfagna}@polito.it

[2]Dept of Mechanical and Manufacturing Engineering, The University of Calgary

2500 University Drive NW, Calgary, AB, T2N 1N4, Canada. E-mail: salvatore.federico@ucalgary.ca

The material properties of a fibre-reinforced biological tissue, be it natural or engineered, depend on a multiplicity of processes that transform the tissue's internal structure in order to optmise its functioning in response to the interactions with the surrounding environment. Among the various processes that induce the structural evolution, a relevant role is played by the reorganisation of the fibre pattern. This, indeed, influences both the mechanical behaviour of the tissue (e.g., the capability of bearing loads) and the interaction of the tissue with the interstitial fluid circulating throughout it.

Studying the reorientation of the fibres is important for improving the mechanical properties of engineered tissues and, more specifically, for obtaining the fibre pattern characterising the original, healthy tissues. This is true, for instance, for *in vitro* cartilage [1], which is meant to replace the original tissue when its mechanical properties are deteriorated by injuries or degenerative diseases such as osteoarthritis [1].

After reviewing some selected works on the subject, e.g., [1, 2, 3, 4, 5, 6], the model of fibre reorientation recently proposed in [7] is summarised and presented. The main assumption of the considered model is that the collagen fibres are oriented statistically according to a probability density distribution compatible with experimental data [8]. The probability distribution is assumed to depend on a set of parameters (referred to as "remodelling parameters") that vary with time and on the material point with which they are associated. Their evolution is governed by appropriate dynamic equations [4, 5, 6, 7], and is deduced on the basis of the hypothesis that the fibres vary their orientation both passively, i.e., as a consequence of the deformation drift, and actively. The active reorientation is modelled by introducing a suitable *remodelling energy* that accounts for the interaction with the deformation, and is assumed to comprise a term of the Ginzburg-Landau type, and a term depending on the gradient of the mean angle of the probability density.

By minimising the remodelling energy in the absence of deformation, the histological profile of fibre alignment in the tissue's undeformed configuration is retrieved. Afterwards, the fibre reorientation is studied together with deformation in response to a given loading protocol. In this case, it can be observed that the pattern of fibre alignment obtained under loading resembles the one characterising the osteoarthritic tissue. This result could be a point of departure for further studies in this direction.

References

[1] Wilson, W., Driessen, N.J.B., van Donkelaar, C.C., Ito, K.: Prediction of collagen orientation in articular cartilage by a collagen remodeling algorithm. *Osteoarthritis and Cartilage*, **11**, 1196–1202 (2006).

[2] Driessen, N.J.B., Peters, G.W.M., Huyghe, J.M., Bouten, C.V.C., Baaijens, F.P.T.: Remodelling of continuously distributed collagen fibres in soft connective tissues. *Journal of Biomechanics*, **36**, 1151–1158 (2003).

[3] Olsson, T., Klarbring, A.: Residual stresses in soft tissue as a consequence of growth and remodeling: application to an arterial geometry. *European Journal of Mechanics A/Solids*, **27**, 959–974 (2008).

[4] Baaijens, F., Bouten, C., Driessen, N.: Modeling collagen remodeling. *Journal of Biomechanics*, **43**, 166–175 (2010).

[5] Grillo, A., Federico, S., Wittum, G.: Growth, mass transfer, and remodeling in fiber-reinforced, multi-constituentmaterials. *International Journal of Non-Linear Mechanics*, **47**, 388–401 (2012).

[6] Grillo, A., Wittum, G., Tomic, A., Federico, S.: Remodelling in statistically oriented fibre-reinforced materials and biological tissues. *Mathematics and Mechanics of Solids*, **20(9)**, 1107–1129 (2015).

[7] Grillo, A., Carfagna, M., Federico, S.: An Allen-Cahn approach to the remodelling of fibre-reinforced anisotropic materials. *Submitted*.

[8] Federico, S., Grillo, A., La Rosa, G., Giaquinta, G., Herzog, W.: A transversely isotropic, transversely homogeneous microstructural-statistical model of articular cartilage. *Journal of Biomechanics*, **38**, 2008–2018 (2005).