Plasticity and remodelling in fibre-reinforced tissues

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This work addresses the description of fibre-reinforced, soft, hydrated biological tissues featuring a statistical orientation of reinforcing fibres [1, 2]. Our aim is to study the mechanical behaviour of a tissue of this type by highlighting the coupling among deformation, plastic distortions, reorganisation of the fibre pattern, and evolution of the interstitial fluid. To this end, we provide a constitutive framework within which the tissue is regarded as a transversely isotropic material characterised by hyperelastic behaviour from a relaxed, natural state. Furthermore, we assume that two types of dissipative processes occur, both contributing to alter its internal structure. These consist, respectively, of the development of anelastic distortions, and the reorientation of the collagen fibres. The first process is meant to represent the irreversible and incompatible deformations that may be triggered off by injuries, traumatic events, or the rearrangement of the cells' adhesion properties [3, 4]. The second process, instead, is thought of as the both active and passive response of the tissue's fibre network to the mechanical stimuli to which it is exposed.

Whereas the tissue's deformation and the flow of the interstitial fluid are described by the equations characterising the standard biphasic model of hydrated tissues [5], the equations determining the anelastic distortions and the fibre reorientation are obtained within the theory of two-layer dynamics put forward in [6]. In particular, following [7, 8], the reorientation of the fibres is modelled through the evolution of the parameters characterising a given probability density, which measures the probability of finding a fibre aligned along a given direction of space at a given material point.

References

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