Preliminary results on the use of the Darcy-Brinkman equation in fibre-reinforced porous media of biological interest

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Articular cartilage is a biological tissue often described as a multi-constituent material [1, 2]. Its main constituents are an interstitial fluid and a porous matrix of proteoglycans reinforced by collagen fibers.

The material response of articular cartilage is determined by the complex interactions occurring among the interstitial fluid, the matrix and the fiber network. Usually the hydraulic and mechanical behavior of articular cartilage is investigated by performing compression tests both *in vitro* and *in silico* [2]. The majority of the mathematical models employed in numerical simulations is based on Darcy's law. Hence, fluid flow is linearly related through permeability to the pressure gradient applied to the tissue. Although Darcy's law permits to obtain reasonable results in several cases of interest, it is unable to explicitly resolve the dependence of fluid velocity on space, and it cannot account for boundary effects [3].

To perform a more detailed description of the flow, in this work we provide a model of articular cartilage based on the Darcy-Brinkman equation [4]. This implies to introduce a viscous stress tensor depending on the fluid velocity gradient through an effective viscosity, and, above all, to determine the fluid velocity by solving a partial differential equation.

We expect that our results, although being preliminary, may be extended to model other porous media characterized by complex internal structure.

References

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