Second order effects in the problems of equilibrium and stability of nonlinearly elastic body with microstructure

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A new wave of interest to the models of continuum mechanics based on the hypothesis of kinematic independence of the displacement and rotation fields, entered into the usage by Cosserat brothers more than a hundred years ago, is associated primarily with the needs of nanomechanics. Such models of the elastic behavior can be of use to take into account the structure of the material and some surface effects; furthermore they are relevant in many areas of biomechanics. The main task of this study was to find such statements of more or less classical problems for nonlinearly elastic bodies that have solutions significantly different in the classical theory of elasticity and in the theory of the Cosserat continuum. In particular, such situations can be the basis for the development of experimental techniques for the identification of parameters of constitutive relations of media with microstructure and for the verification of the models used.

We used semi-inverse method of nonlinear elasticity to study three problems, namely the problem of a nonlinear torsion of the circular cylinder, the problem of pure bending of the panel with rectangle cross-section, and the problem of the cylinder stretching. The problems differ in variants of semi-inverse representation of the displacements and in functions of the specific potential energy of the Cosserat medium, though in all cases the semi-inverse representation contains two functions to be defined (the radial displacement and the micro-rotation angle of the particles of the body), depending only on the one scalar parameter.

The first part of the study was focused on the second-order effects. In the problem of torsion it was well known Pointing effect, i.e. length change of the twisting cylinder in the absence of longitudinal force. In the case of bending the variation in the thickness of the panel was investigated. By direct numerical computations and asymptotical analysis it was shown that for some models describing large strains of micropolar media there exists not quantitative but also qualitative difference in the behavior of solutions of problems for classical and Cosserat continuum.

The analysis of the stability was performed in the vicinity of so called material instability zones, e.g. in the neighborhood of the point of a maximum on the stretching diagram. The similar maximum point following by the falling region can exist at the bending and torsion diagrams also. This falling region is usually treated as the onset of the instability. At the same time it is possible to show that there are some parameters of the constitutive relation and geometrical sizes of the sample when the deformation along the whole falling region or its' considerable part is stable. The performed analysis showed that material constants, or more exact – relations between them, make appreciable quantitative and qualitative impact on a stability zone and its size. By means of selection of these constants it is possible to achieve both essential expansion of this zone and its elimination.

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