Internal variables associated with microstructure

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Prediction of the response of microstructured materials on an external loading can be achieved by means of various methods. In (quasi)statics, homogenization methods are suitable in most situations, but this is not the case for functionally graded materials, e.g. Strain gradient models are quite sufficient if only the influence of a microscale length is taken into account. The most general approach is provided by generalized continuum theories, which include microdeformation into consideration. One more possibility is the introduction of internal variables for the description of microstructure.

In the paper, we compare different descriptions of microstructured solids on the simple example of wave propagation in the one-dimensional setting. In the classical continuum mechanics the existence of a microstructure is neglected. Thus, the classical wave equation needs to be modified to include the observed dispersive effects due to the microstructure. We consider modifications of the wave equation which follow from homogenization, continualization of lattice models, and from generalized continuum theories. The linear version of the Boussinesq equation for elastic crystals, the Love-Rayleigh equation for rods accounting for lateral inertia, the Maxwell-Rayleigh model of anomalous dispersion, etc., are compared with dispersive wave equations obtained by means of single and dual internal variables. The similarity and distinction of the models are analyzed and emphasized.