C^1–CONTINUOUS APPROXIMATIONS USING HERMITE FINITE ELEMENTS FOR SECOND GRADIENT ELASTIC PROBLEMS

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Abstract

Materials with intrinsic micro- or nano-structure may show size-dependent material behavior, which is reflected, e. g., by a stiffer elastic response to external forces when the size of the material body is reduced. In order to account for the so-called size effect higher-order continuum models are frequently used, which go beyond the limits of conventional CAUCHY continuum. In this work, a modified strain gradient theory of elasticity for isotropic materials is presented [1, 2, 3]. The model is applicable to so-called non-simple materials of the gradient type [4], applicable to, for example, polymers at a small scale. The concept of rational analytical mechanics is extened up to higher-order terms for the spatial dependency of material points of a body in a finite neighborhood. This leads to some modifications in the strain energy formulation and three additional material length scale parameters need to be introduced. The strain energy function of such continuum models involve second gradients of displacements. Some boundary value problems are solved analytically, as for example the problem of static beam or plate bending. Numerical solutions are derived by means of a finite element approach using the GALERKIN method. A global C^1-continuity of the displacement field is achieved by using so-called HERMITE finite elements [5], in order to assure a linkage of the gradients of the element's basis functions. The element stiffness matrix, as well as the global stiffness matrix for one and two dimensional problems are developed and solved numerically. EULER-BERNOULLI beam bending, KIRCHHOFF-LOVE plate bending and a general plane strain condition of higher order formulations are analyzed. Convergence of the FE-strategy is achieved, C^1-continuity is shown, and the incorporation of the size effect can be presented. Experiments on bending stiffnesses of differently sized micro cantilevers, made of the polymer SU-8, have been performed using an atomic force microscope [6]. The results are compared to the numerical as well as semi-analytical solutions and a higher-order material parameter is identified from the experiments.

References

[1] Mindlin R.D., Eshel N.N.: *On first strain-gradient theories in linear elasticity*. International Journal of Solids and Structures, **4**, pp. 109–124 (1968)

[2] Fleck N.A., Hutchinson J.W.: *Strain gradient plasticity*. In: Hutchinson J.W., Wu T.Y. (Eds.), Advances in Applied Mechanics, **33**, Academic Press, New York, pp. 295–361 (1997)

[3] Yang F., Chong C.M., Lam D.C.C., Tong P.: *Couple stress based strain gradient theory for elasticity*. International Journal of Solids and Structures, **39**(10), pp. 2731–2743 (2002)

[4] Eringen A.C.: *Nonlocal Continuum field theories*. Springer-Verlag, New York (2010)

[5] Gunakala S.R., Comissiong D.M.G., Jordan K., Sankar A.: *A Finite Element Solution of the Beam Equation via MATLAB*. International Journal of Applied Science and Technology, **2**(8), pp. 80–88 (2012)

[6] Liebold C., Müller W.H.: *Measuring material coefficients of higher gradient elasticity by using AFM techniques and Raman-spectroscopy*. Advanced Structured Materials **22**, pp. 255–271 (2013)

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