Electromagnetic microcontinua and Maxwell's equations in matter

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The common microcontinuum theories of electromagnetic elastic media rely on the extension of the mechanical micromorphic model to electromagnetic coupling [1,2]. Maxwell's equations are added to mechanical balance laws and classical constitutive equations for polarization and magnetization are generalized to account for micro-deformations. The resulting model requires complex constitutive assumptions involving electric and magnetic fields beside microscopic strain measures.
An alternative approach can be proposed on the basis of the following physical reasons.
A consistent electromagneto-elastic microcontinuum model should put mass and bound charge microdensities on the same ground in dealing with coupling effects. Moreover, at least for certain spatial scales, micro-deformation implies a corresponding variation of spatial distribution of bound charges so that electric and magnetic multipoles can be described by the same micro-displacements connected to mechanical deformations. According to classical electromagnetic theories, these multipoles play an essential role in the derivation of Maxwell's equations in matter, yielding a proper definition of polarization and magnetization (see for example [3,4] and references therein).
A continuum micromorphic model for electro-elastic media, based on electric multipole densities, discussed in some previous works (see for example [5,6]) is here extended to deal with electric conductors. In this model polarization **P** and magnetization **M** are obtained from the derivation of macroscopic Maxwell's equations in terms of multipoles which, in turn, satisfy suitable evolution equations [7,8]. Only bound charges contribute to **P** and **M**, and a convective term arises in electric current owing to microdeformation. Charge carriers are here considered as a continuum superimposed to the microstructured conductor and the case of a rigid polarized conductor is discussed. One dimensional plane waves are also analyzed to show the effect of conductivity and polarization on the dispersion equation.

References

[1] Eringen, A.C., *Microcontinuum Field Theories*, voll. I, II. Springer-Verlag, New York (1999).
[2] Lee, J.D., Chen, Y., Eskandarian, A., "A micromorphic electromagnetic theory". Int. J. Solids Struct. **41**, 2099-2110 (2004).
[3] Jackson, J.D., *Classical Electrodynamics*, 3rd ed., John Wiley, New York (1999).
[4] Maugin, G.A., *Continuum Mechanics of Electromagnetic Solids*, North Holland, Amsterdam, (1988).
[5] Romeo, M., "Micromorphic continuum model for electromagnetoelastic solids", *J. App. Math Phys (ZAMP)*, **62**, 513-527 (2011).
[6] Romeo, M., "Polarization in dielectrics modeled as micromorphic continua", *J. App. Math Phys (ZAMP)*, **66**, 1233-1247 (2015)
[7] Romeo, M., "A Microstructure continuum approach to electromagneto-elastic conductors", *Cont. Mech. Thermodyn.*, **28**, 1659-1670 (2016)
[8] Romeo, M., "Electromagnetic field-current coupling in rigid polarized conductors", *Math. Mech. Sol.*, in press, DOI:10.1177/1081286516666404.