Symmetry analysis and conservation laws of fully non-linear wave models in fiber-reinforced anisotropic incompressible hyperelastic solids

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Many composite materials, including biological tissues, are modeled as non-linear elastic materials reinforced with elastic fibers. In the current paper, the full sets of dynamic equations for finite deformations of incompressible hyperelastic solids containing a single fiber family are considered. The models considered in this contribution are appropriate for the mathematical description of certain aspects of the behaviour of biological membranes and similar structures. Finite-amplitude wave propagation ansätze compatible with the incompressibility condition are employed for a generic fiber family orientation, and corresponding non-linear and linear wave equations are derived. It is shown that for a certain class of constitutive relations, the fiber contribution vanishes when the displacement is independent of the fiber direction. Point symmetries of the derived wave models are classified with respect to the material parameters and the angle between fibers and the wave propagation direction. Examples of exact time-dependent solutions are provided in several physical situations, including the evolution of pre-strained configurations and traveling waves. This framework is next extended to incorporate a viscoelastic deformation of the matrix, thereby considering the context of hyperviscoelasticity. Equivalence transformations are used to simplify the resulting wave equations and to reduce the number of parameters. Local conservation laws and global conserved quantities of the models are systematically computed and discussed, along with other related mathematical properties. Sample numerical solutions are presented.