Inaugural Summer School
of the CNRS International Associate Laboratory Coss&Vita
on Mechanics of generalized continua and their applications to engineering materials and structures
Arpino, July 20-26th 2015

Titles and abstract of proposed Courses
http://www.f2m.cnrs-bellevue.fr/spip.php?article781
http://www.memocsevents.eu/wordpress/cossevita/inaugural-summer-school-2/

Microstructured higher gradient continuum models for complex multiscale-multiphysics materials

Francesco dell'Isola

Università degli Studi di Roma *La Sapienza*,
Dipartimento di Ingegneria Strutturale e Geotecnica,
Via Eudossiana, 18 - 00184 Roma

francesco.dellisola@uniroma1.it

http://www.fdellisola.it/

Complex Composite materials may have a complex behaviour at macro-scale which is usually related to their micro-structure. In particular composites may be endowed with a multiscale structure and may show at micro-level high contrast in physical and geometrical properties. As happens in living tissues, smart composites may be able to change their constitutive equations by means of self-controlling processes, which may be driven by mechanically suitably produced stimuli.

Some numerical, theoretical and experimental results which were recently obtained will be presented showing that a class of fibrous fabrics must be modelled by means of second gradient continua (at least) and that one can conceive to design new and efficient metamaterials whose performances can be really exotic. In particular some microstructured fabrics constituted by fibres having bending stiffness (pantographic sheets) and nearly-inextensible are carefully studied using microscopic and macroscopic models. These examples show that: i) macro-models cannot belong to the class of Cauchy first gradient continua ii) some fabrics whose micro-structure is really simple may have a very complex macro-behaviour, iii) the dynamical response of pantographic sheets can be really unexpected and iv) some delicate experimental set-ups are needed to measure their main physical properties.

A digression on the concept of generalised contact forces in higher gradient continua and on the boundary conditions naturally arising in the theory of generalised continua will be necessary to consistently present the obtained result. This digression shows some of the the limits of standard continuum mechanics as conceived by Cauchy and motivates the conceptual effort (based on the Lagrangian Principle of Virtual Work) which has been started by Gabrio Piola, continued by Toupin, Mindlin and Germain and recently re-started by several research groups to found the correct conceptual frame for generalised continuum mechanics.
Some applications to wave propagation in complex materials are presented including some results about propagation in 2D pantographic structures which seem to indicate the existence of some solitary waves. In particular some micro-structures including piezoelectric actuators interconnected by optimal circuits may be considered. The concept of optimal energy transduction is very fruitful for instance in vibration and acoustic suppression.

The postulations à la D'Alembert and à la Cauchy for higher gradient continuum theories are equivalent: a review of existing results.

Pierre Seppecher

Institut de Mathématiques de Toulon, (IMATH),
Universite de Toulon et du Var, U.F.R. des Sciences et Techniques,
Avenue de l'Université,
BP 132, 83957 La Garde, Cedex.
Tel: 04 94 14 20 25 Fax: 04 94 14 26 33, e-mail: seppecher@univ-tln.fr
http://seppecher.univ-tln.fr

In order to found continuum mechanics two different postulations have been used in the literature. The first of them was introduced by Lagrange and Piola and starts by postulating how the work expended by internal interactions in a body depends on the virtual velocity field and its gradients. Then, by using the theorem of divergence, a representation theorem is found for the volume and contact interactions which can be exerted at the boundary of the considered body by the external world. This first method assumes as fundamental the notion of internal work, regards stress tensors as dual of virtual displacements and their gradients and deduces as a derived one the concept of contact interactions and produces their representation in terms of stresses by means of integration by parts. The second method starts by postulating the type of contact interactions which can be exerted on the boundary of every (suitably) regular part of a body and then proceeds by proving the existence of stress tensors from a balance-type postulate. This second method was followed by Cauchy using the celebrated tetrahedron argument. In this paper we review some relevant literature in the subject and, in particular, we discuss how the two postulations can be reconciled in the case of higher gradient theories. Finally we underline the importance of the concept of contact surface, edge and wedge sorder forces.

References

How contact interactions may depend on the shape of Cauchy cuts in Nth gradient continua: approach "à la D'Alembert" F dell'Isola, P Seppecher, A Madeo, Zeitschrift für angewandte Mathematik und Physik 63 (6), 1119-1141, 2012.

Micromorphic vs. gradient approaches to size-dependent elastoplasticity of materials

Samuel Forest
Mines ParisTech CNRS
Materials Center UMR 7633
BP 87
91003 Evry France
samuel.forest at ensmp.fr

The objective of this course is to set the fundamentals of Eringen and Mindlin's micromorphic theory and to extend it to the nonlinear behaviour of materials, especially the plasticity of metals. The micromorphic approach to gradient elastoplasticity and damage is a powerful method to address the continuum modelling of size effects in engineering materials, like grain size effects in polycrystals and strain localization phenomena in porous metals. It allows for straightforward finite element implementation using standard computational mechanics methods. The following steps will be followed:

* Kinematics and dynamics of micromorphic media

* Relation to gradient theories

* Thermomechanics of elastoplastic micromorphic media

* Application to strain localization phenomena in metals: analytical solutions and results of finite element simulations.

References to be downloaded from
http://matperso.mines-paristech.fr/People/samuel.forest/Publications/

S. Forest and R. Sievert, *Elastoviscoplastic constitutive frameworks for generalized continua* , Acta Mechanica, vol. 160, pp. 71-111, 2003.

S. Forest, *Questioning size effects as predicted by strain gradient plasticity*, Journal of the Mechanical Behavior of Materials, vol. 22, pp. 101-110, 2013. doi:10.1515/jmbm-2013-0015

S. Forest, *Micromorphic approach for gradient elasticity, viscoplasticity and damage* , ASCE Journal of Engineering Mechanics, vol. 135, pp. 117-131, 2009.

S. Forest, A. Bertram, *Formulations of strain gradient plasticity*. In *Mechanics of generalized continua,* edited by Holm Altenbach, G.A. Maugin and V. Erofeev, Advanced Structured Materials vol. 7, Springer, pp. 137-150, 2011.

M. Mazière and S. Forest, *Strain gradient plasticity modeling and finite element simulation of Lüders band formation and propagation,* Continuum Mechanics and Thermodynamics, vol. 27, pp. 83-104, 2015. doi:10.1007/s00161-013-0331-8

Inner resonance media - Principle and example

Claude Boutin
Ecole Nationale des Travaux Publics de l'Etat – ENTPE
Département Génie Civil et Bâtiment
Rue Maurice Audin, 69518 Vaulx-en-Velin Cedex

E-mail : claude.boutin@entpe.fr

http://www.entpe.fr/internet/contenu/departements/genie_civil_batiment/laboratoire_genie_civil_et_batiment_lgcb/dynamique_auscultation_controle/equipe_dynamique_auscultation/claude_boutin

This course deals with inner resonance media, in which dynamic phenomena coexist at both micro and macro scales. Such a "co-dynamics" regime is only possible in heterogeneous media, currently named "metamaterials". These materials are of prime interest for their properties that seems impossible to reach with classical materials. Indeed, whatever the physical nature of the inner resonance, the description strongly departs from standard dynamics. The key discrepancy is that the effective parameters are frequency dependent and can take negative values in a frequency range related to the inner-resonance frequency.

The aim of the course is twofold. First, to translate the physics that lead to inner resonance in composite media into design rules expressed in terms of morphology and/or specificity of the mechanical parameters of the constituents. Second, to apply this approach to different physics and materials and to derive their dynamic features by homogenization.

The course is structured as follows. First some generalities on the homogenization method are exposed. Then, we address successively the cases of :

- Highly contrasted elastic bi-composites for which the inner resonance results in effective negative inertia around specific frequency band. This induces band-gaps at large wavelength.

- Reticulated media made of bars of higher stiffness in compression and than in bending. The local bending resonance leads to distinct but similar effects than in bi-composites.

- Double porosity media saturated by gas : the co-dynamic regime leads to a frequency dependent compressibility driven by the viscosity. Hence, a significant dissipation appears without band-gap.

- Porous media with inner resonators. Here, the co-dynamics is reached via a geometrical contrast. Around resonance, the resonator brought a negative contribution to the effective gas stiffness inducing a broad band gap along with strongly dispersed waves.

The conclusion emphasis on the similarities of the results related to different physics, showing that inner resonance media requires a highly contrasted microstructure. Some experimental evidence of inner-resonance will also be presented and discussed.

Some References

Sanchez-Palencia E (1980), Non-Homogeneous Media and Vibration theory, Springer Berlin.

Auriault JL & Bonnet G (1985), Arch. Mech. 37(4-5), 269-284

Auriault JL & C.Boutin C (2012) I.J.S.S (49), 3269-3281

Chesnais C, Boutin C, Hans S (2012) J.A.S.A. (132), 4 ; 2873-2886

"Dimension reduction" procedures to extract models of plates/shells/fibers from 3D theories, including elasticity, plasticity, diffusion, thermoelasticity.

David Steigmann

6133 Etcheverry Hall, Mailstop 1740
University of California at Berkeley
Berkeley, CA 94720-1740
dsteigmann@berkeley.edu

http://www.me.berkeley.edu/faculty/steigman/

One of the most interesting problems of contemporary mathematical physics consists in the formulation of homogenised models simplifying more complex ones, in order to get for instance numerical computations which are less demanding. Of course also qualitative semi-analytical studies become more feasible when such simplified models are introduced.

Homogenised models can have the same dimension as the refined ones: however when one or two characteristic lengths can be neglected in the homogenised model the model decreases its dimension under the homogenisation procedure.
In this case we talk about “dimension reduction”: this what happens for the classical Euler-Bernoulli, Timoshenko beam theories or for the Kirchhoff- Love plate theories. The resulting reduced models are invariably of the 'higher gradient’ type: their deformation energy depends on higher gradient of displacement fields or of the other kinematical descriptors.

The lectures will focus in particular on Cosserat elasticity (fibers) but the treated topics will include also some basic and applicable differential geometry, some aspects of rheology of lower dimensional continua, multi physics phenomena including e.g. electromagnetics couplings.

Tensor decompositions: Application to classical and generalized continua

N. Auffray
Laboratoire de Modélisation et Simulation Multi Echelle
Equipe de Mécanique
5, Boulevard Descartes
77454, Marne-la-Vallée Cedex 2
nicolas.auffray@univ-paris-est.fr

In Continuum Mechanics tensors are used to describe the mechanical state of the matter and its linear behavior. A well-known example is the Hooke Law in which the second order strain and stress tensors are related through the use of the elasticity tensor which is fourth-order.
This "simple" relation contains meaningful information about the physics which is encoded by linear elasticity. To reveal this information the elasticity tensor has to be decomposed, and its "elementary" components studied. As a consequence, and among other results, it can be concluded that:
– there exists only 8 type of anisotropic elastic behavior, all of them being achiral;
– hexagonal lattice are elastically transverse isotropic.
This approach, which can naturally be extended to higher order tensors, constitutes a valuable tool to explore the capability of different generalized continua to describe different physical phenomena. This lecture will be devoted to the introduction of this method, which is based on group representation theory and which is well-known in the field of condensed matter physics.

References

Auffray, N., Dirrenberger, J., & Rosi, G. (2015). A complete description of bi-dimensional anisotropic strain-gradient elasticity. *International Journal of Solids and Structures*, Accepted.

Forte, S., & Vianello, M. (1996). Symmetry classes for elasticity tensors. *Journal of Elasticity*, *43*(2), 81-108.

Jerphagnon, J., Chemla, D., & Bonneville, R. (1978). The description of the physical properties of condensed matter using irreducible tensors. *Advances in Physics*, *27*(4), 609-650.

Olive, M., & Auffray, N. (2013). Symmetry classes for even-order tensors. *Mathematics and Mechanics of Complex Systems*, *1*(2), 177-210.

The Bending Gradient Theory for architectured plates

Arthur Lebée
Laboratoire Navier
Ecole des Ponts Paristech - IFSTTAR - CNRS UMR 8205
6-8 Av. B. Pascal
77420 Champs-sur-Marne, FRANCE
http://navier.enpc.fr/LEBEE-ARTHUR
arthur.lebee@enpc.fr

The classical theory of plates, known also as Kirchhoff-Love plate theory is based on the assumption that the normal to the mid-plane of the plate remains normal after transformation. This theory is also the first order of the asymptotic expansion with respect to the thickness. Thus, it presents a good theoretical justification and was soundly extended to the case of periodic plates. It enables to have a first order estimate of the macroscopic deflection as well as local stress fields. In most applications the first order deflection is accurate enough. However, this theory does not capture the local effect of shear forces on the microstructure because shear forces are one higher-order derivative of the bending moment in equilibrium equations.
Because shear forces are part of the macroscopic equilibrium of the plate, their effect is also of great interest for engineers when designing structures. However, modeling properly the action of shear forces is still a controversial issue.
Revisiting the approach from Reissner directly with laminated plates, it appears that the transverse shear static variables which come out when the plate is heterogeneous are not shear forces but the full gradient of the bending moment. Using conventional variational tools, it is possible to derive a new plate theory, called Bending-Gradient theory. This new plate theory is considered as an extension of Reissner's theory to heterogeneous plates which preserves most of its simplicity. Originally designed for laminated plates, it is also extended to in-plane periodic plates using averaging considerations. The lecture will be illustrated with application to strongly heterogeneous plates such as cellular sandwich panels or periodic space frames.

References.

Lebée, A.; Sab, K., A Bending-Gradient model for thick plates. Part I: Theory, *International Journal of Solids and Structures,* vol. 48, pp. 2878-2888, 2011.

Lebée, A.; Sab, K., A Bending-Gradient model for thick plates, Part II: Closed-form solutions for cylindrical bending of laminates, *International Journal of Solids and Structures*, vol. 48, pp. 2889-2901, 2011.

Lebée, A.; Sab, K., Transverse shear stiffness of a chevron folded core used in sandwich construction, *International Journal of Solids and Structures*, vol. 47, pp. 2620-2629, 2010.

Generalized continuum theories and applications to metamaterials exhibiting band-gaps and to fibrous composite reinforcements

Angela Madeo
Université de Lyon-INSA (Institut National des Sciences Appliquées)
Laboratoire de Génie Civil et Ingénierie Environnementale (LGCIE)
Bâtiment Coulomb, 69100 Villeurbanne, France
angela.madeo@insa-lyon.fr

The micro-structure of materials is an essential feature for the design of engineering structures with improved performances. It is conceivable, at the present stage of knowledge and technology, to direct a consistent scientific effort towards the conception of micro-structured materials showing exotic properties which may be beneficial for the functioning of engineering structures and for their optimization. Actually, engineering structures designed using micro-structured materials (also called metamaterials or architectured materials) may show very interesting mechanical properties such as light weight, improved stiffness, easy forming processes and so on. Moreover, such materials could also be used for innovative applications in the field of vibration control and stealth technology. In fact, architectured materials are good candidates for the conception of wave screens and wave absorbers since they may show peculiar properties with respect to wave propagation. Many scientific challenges related to the application of generalized continuum theories to the characterization and conception of high-performance metamaterials can be identified. In this work, we identify two main potential fields of applications of generalized continuum theories, namely: wave propagation in metamaterialsmechanical behavior of fibrous composite reinforcements
It is known that some materials like phononic crystals and lattice structures (see e.g. [1]), granular assemblies with defects (see e.g. [2]) and composites ([3]) can inhibit wave propagation in particular frequency ranges (band-gaps). We propose to use the newly developed relaxed micromorphic model presented in [4,5] to study wave propagation in microstructured materials which exhibit frequency band-gaps. The proposed relaxed model only counts 6 constitutive parameters and is fully able to account for the effect of microstructure on the macroscopic mechanical behavior of considered media. The limited number of constitutive parameters makes possible the future conception of direct and indirect measurements on real materials exhibiting frequency band-gaps.
The second promising field of application of generalized continuum theories is that of the study of the mechanical behavior of woven fibrous composite reinforcements. Such metamaterials are constituted by two order of fibers which have very high elongation stiffness, but very low shear stiffness (see e.g. [7]). This strong contrast in the mechanical properties of the mesostructure is such that the homogenized material must necessarily be described at least in the framework of second gradient theories (see [8-11]).

[1] Vasseur J.O. and Deymier P.A. et al. Experimental and theoretical evidence for the existence of absolute acoustic band gaps in two-dimensional solid phononic crystals. Pysical Review Letters,, 86(14):3012–3015, 2001.
[2] Kafesaki M., Sigalas M.M., and GarcíaN. Frequency modulation in the transmittivity of wave guides in elastic- wave band-gap materials. Physical Review Letters, 85(19):4044–4047, 2000.
[3] Vasseur J.O. and Deymier P.A et al. Experimental evidence for the existence of absolute acoustic band gaps in two-dimensional periodic composite media. J. Phys. Condens. Matter, 10:6051, 1998.
[4] Neff P. and Ghiba I.D., Madeo A., Placidi L., and Rosi G. A unifying perspective: the relaxed linear micro- morphic continuum. arXiv:1308.3219 Submitted to Continuum Mechanics and Thermodynamics, 2013.
[5] Ghiba I.D. andNeff P., Madeo A., Placidi L., and Rosi G. The relaxed linear micromorphic continuum: existence, uniqueness and continuous dependence in dynamics. Submitted to

Mathematics and Mechanics of Solids, arXiv:1308.3762v1 [math.AP], 2013.
[7] Charmetant A. and Boisse P. Orliac J.G., Vidal Sallée E. Hyperelastic model for large deformation analyses of 3d interlock composite preforms. Composites Science and Technology, 72 1352–1360:1352–1360, 2012.
[8] Alibert J.-J., Seppecher P., and dell'Isola F. Truss modular beams with deformation energy depending on higher displacement gradients. Math. Mech. Solids, 8(1):51–73, 2003.
[9] Seppecher P., Alibert J.-J., and dell'Isola F. Linear elastic trusses leading to continua with exotic mechanical interactions. Journal of Physics: Conference Series, 319(1):012018, 2011.
[10] Ferretti M., Madeo A., dell'Isola F., and Boisse. Modelling the onset of shear boundary layers in fibrous composite reinforcements by second gradient theory. ZAMP, 65(3):587–612, 2014.
[11] A. Madeo, M. Ferretti, F. dell'Isola, P. Boisse (2015). "Thick fibrous composite reinforcements behave as spe- cial second-gradient materials: three-point bending of 3D interlocks". Zeitschrift für angewandte Mathematik und Physik (ZAMP), DOI: 10.1007/s00033-015-0496-z

Gradient damage models and brittle fracture: the variational perspective

Corrado Maurini

Institut Jean Le Rond d'Alembert, Université Pierre et Marie Curie

Tour 55/65, 4 Place Jussieu, 75252 Paris
corrado.maurini@upmc.fr

The classical theory of fracture mechanics developed in the last century is very successful in stating the non-propagation conditions for pre-existing cracks, or to study their propagation along pre-assigned paths.
However, the theoretical modelling and the predictive numerical simulation of crack nucleation and the emergence of complex crack patterns is still an open research subject. In the last years, variational approaches to fracture opened new perspectives, by formulating fracture mechanics as an energy minimisation problem where the crack path is treated as a genuine unknown. This lecture will briefly introduce to the variational approach to fracture and discuss the use of gradient damage models as a phase-field regularised formulation of brittle fracture. Reporting the results of theoretical analyses and numerical experiments, we will show how similar models can retrieve crack nucleation, crack propagation, and the morphogenesis of complex crack patterns in 2D or 3D.

References

B. Bourdin, J.J. Marigo, C. Maurini, P. Sicsic, Morphogenesis and propagation of complex cracks induced by thermal shocks, Physical Review Letters 112, 014301 (2014)

K. Pham, H. Amor, J.-J. Marigo, C. Maurini, Gradient damage models and their use to approximate brittle fracture, International Journal of Damage Mechanics 20 (4) 618-652 (2011)

H. Amor, J.-J. Marigo, C. Maurini, Regularized formulation of the variational brittle fracture with unilateral contact: numerical experiments, Journal of the Mechanics and Physics of Solids 57 (8), 1209-1229 (2009)

B Bourdin, GA Francfort, JJ Marigo, The variational approach to fracture, Journal of Elasticity 91 (1-3), 5-148

Waves in second gradient materials Transmission and reflection at discontinuity surfaces

Luca Placidi
Ricercatore di Scienza delle Costruzioni, Università Telematica Internazionale Uninettuno
luca.placidi@uninettunouniversity.net

In this lecture reflection and transmission of compression and shear waves at structured interfaces between second- gradient elastic continua is investigated. Two semi-infinite spaces filled with the same second-gradient material are connected through an interface which is assumed to have its own material properties (mass density, elasticity and inertia). Using a variational principle, general balance equations are deduced for the bulk system, as well as jump duality conditions for the considered structured interfaces. The obtained equations include the effect of surface inertial and elastic properties on the motion of the overall system.

In the first part of the lecture general 3D equations accounting for all surface deformation modes (including bending) are introduced. The application to wave propagation is presented in the second part of the lecture. It is based on a simplified 1D version of these equations.

References

Reflection and transmission of plane waves at surfaces carrying material properties and embedded in second-gradient materials, L Placidi, G Rosi, I Giorgio, A Madeo, Mathematics and Mechanics of Solids, 1081286512474016