Electromagnetic microcontinua and Maxwell’s equations in matter

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The common microcontinuum theories of electromagnetic elastic media rely on the extension of the mechanical micromorphic model to electromagnetic coupling [1,2]. Maxwell’s equations are added to mechanical balance laws and classical constitutive equations for polarization and magnetization are generalized to account for micro-deformations. The resulting model requires complex constitutive assumptions involving electric and magnetic fields beside microscopic strain measures.

An alternative approach can be proposed on the basis of the following physical reasons.

A consistent electromagneto-elastic microcontinuum model should put mass and bound charge microdensities on the same ground in dealing with coupling effects. Moreover, at least for certain spatial scales, micro-deformation implies a corresponding variation of spatial distribution of bound charges so that electric and magnetic multipoles can be described by the same micro-displacements connected to mechanical deformations. According to classical electromagnetic theories, these multipoles play an essential role in the derivation of Maxwell’s equations in matter, yielding a proper definition of polarization and magnetization (see for example [3,4] and references therein).

A continuum micromorphic model for electro-elastic media, based on electric multipole densities, discussed in some previous works (see for example [5,6]) is here extended to deal with electric conductors. In this model polarization \( P \) and magnetization \( M \) are obtained from the derivation of macroscopic Maxwell’s equations in terms of multipoles which, in turn, satisfy suitable evolution equations [7,8]. Only bound charges contribute to \( P \) and \( M \), and a convective term arises in electric current owing to microdeformation. Charge carriers are here considered as a continuum superimposed to the microstructured conductor and the case of a rigid polarized conductor is discussed. One dimensional plane waves are also analyzed to show the effect of conductivity and polarization on the dispersion equation.

References